

$$(13) \quad (a) \quad (i) \quad AC^2 = (\sqrt{5})^2 - 1^2$$

$$= 5 - 1$$

$$= 4$$

$$AC = 2$$

$$AD^2 = (\sqrt{10})^2 - 1^2$$

$$= 10 - 1$$

$$= 9$$

$$AD = 3$$

$$\cos p = \frac{2}{\sqrt{5}}$$

$$\cos q = \frac{3}{\sqrt{10}}$$

$$(b) \quad \sin(p+q) = \sin p \cos q + \cos p \sin q$$

$$= \left(\frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} \right) + \left(\frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}} \right)$$

$$= \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{50}}$$

$$= \frac{5}{\sqrt{50}}$$

$$= \frac{5}{\sqrt{25} \sqrt{2}}$$

$$= \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

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$$AC = \sqrt{5-1} = 2$$

$$AD = 2+1 = 3$$

$$\cos P = \frac{2}{\sqrt{5}}$$

$$\cos Q = \frac{3}{\sqrt{10}}$$

$$\sin P = \frac{1}{\sqrt{5}}$$

$$\sin Q = \frac{1}{\sqrt{10}}$$

$$\sin(P+Q) = \sin P \cos Q + \cos P \sin Q$$

$$= \frac{1}{\sqrt{5}} \cdot \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{10}}$$

$$= \frac{3+2}{\sqrt{50}} = \frac{5}{5\sqrt{2}} = \underline{\underline{\frac{1}{\sqrt{2}}}}$$

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)	(i)	• ¹ determine $\cos p$	• ¹ $\frac{2}{\sqrt{5}}$	1
		(ii)	• ² determine $\cos q$	• ² $\frac{3}{\sqrt{10}}$	1

Notes:

1. Where candidates do not simplify the perfect squares see Candidates A and B.

Commonly Observed Responses:

Candidate A - no evidence of simplification

$$\cos p = \frac{\sqrt{4}}{\sqrt{5}} \quad \bullet^1 \times$$

$$\cos q = \frac{\sqrt{9}}{\sqrt{10}} \quad \bullet^2 \boxed{\checkmark 1}$$

Repeated error not penalised twice

Candidate B - simplification in part (b)

$$(a) \cos p = \frac{\sqrt{4}}{\sqrt{5}} \cos q = \frac{\sqrt{9}}{\sqrt{10}} \quad \bullet^1 \checkmark$$

$$\vdots$$

$$(b) \sin(p+q) = \frac{5}{\dots} \quad \bullet^2 \checkmark$$

Roots have been simplified in (b)

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		• ³ select appropriate formula and express in terms of p and q • ⁴ substitute into addition formula • ⁵ evaluate $\sin(p+q)$	• ³ $\sin p \cos q + \cos p \sin q$ • ⁴ $\frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}}$ • ⁵ $\frac{1}{\sqrt{2}}$	3

Notes:

2. Award •³ for candidates who write $\sin\left(\frac{1}{\sqrt{5}}\right) \times \cos\left(\frac{3}{\sqrt{10}}\right) + \cos\left(\frac{2}{\sqrt{5}}\right) \times \sin\left(\frac{1}{\sqrt{10}}\right)$. •⁴ and •⁵ are unavailable.

3. For any attempt to use $\sin(p+q) = \sin p + \sin q$, •⁴ and •⁵ are unavailable.

4. At •⁵, accept answers such as $\frac{5}{\sqrt{50}}$ or $\frac{5}{5\sqrt{2}}$ but not $\frac{5}{\sqrt{5} \times \sqrt{10}}$.

5. At •⁵, the answer must be given as a single fraction.

6. Do not penalise trigonometric ratios which are less than -1 or greater than 1 .

Commonly Observed Responses: