

(11)

$$\int_0^{\frac{\pi}{9}} \cos\left(3x - \frac{\pi}{6}\right) dx$$

$$= \left[\sin\left(3x - \frac{\pi}{6}\right) \times \frac{1}{3} \right]_0^{\frac{\pi}{9}}$$

$$= \left(\frac{1}{3} \sin\left(3\left(\frac{\pi}{9}\right) - \frac{\pi}{6}\right) \right) - \left(\frac{1}{3} \sin\left(3(0) - \frac{\pi}{6}\right) \right)$$

$$= \left(\frac{1}{3} \times \sin\left(\frac{\pi}{3} - \frac{\pi}{6}\right) \right) - \left(\frac{1}{3} \sin\left(-\frac{\pi}{6}\right) \right)$$

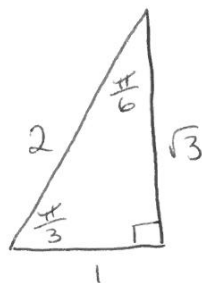
$$= \left(\frac{1}{3} \sin\left(\frac{\pi}{6}\right) \right) - \left(\frac{1}{3} \sin\left(-\frac{\pi}{6}\right) \right)$$

$$= \left(\frac{1}{3} \times \frac{1}{2} \right) - \left(\frac{1}{3} \times \left(-\frac{1}{2}\right) \right)$$

$$= \frac{1}{6} - \left(-\frac{1}{6}\right)$$

$$= \frac{2}{6}$$

$$= \frac{1}{3}$$



Question			Generic scheme	Illustrative scheme	Max mark
11.			•¹ start to integrate •² complete integration •³ substitute limits •⁴ evaluate integral	•¹ $\sin\left(3x - \frac{\pi}{6}\right) \dots$ •² $\dots \times \frac{1}{3}$ •³ $\left(\frac{1}{3} \sin\left(3 \times \frac{\pi}{9} - \frac{\pi}{6}\right)\right) - \left(\frac{1}{3} \sin\left(3 \times 0 - \frac{\pi}{6}\right)\right)$ •⁴ $\frac{1}{3}$	4

Notes:

- Where candidates make no attempt to integrate or start to integrate individual terms within the bracket or use another invalid approach eg $\sin\left(3x - \frac{\pi}{6}\right)^2$ or $\int \cos(3x) - \cos\left(\frac{\pi}{6}\right) dx$, award 0/4.
- Do not penalise the inclusion of '+c' or the continued appearance of the integral sign after •¹.
- Candidates who work in degrees from the start cannot gain •¹. However, •², •³ and •⁴ are still available.
- ¹ may be awarded for the appearance of $\sin\left(3x - \frac{\pi}{6}\right)$ in the first line of working, however see Candidates B and D.
- ⁴ is only available where candidates have considered both limits within a trigonometric function.
- Where candidates use a mixture of degrees and radians, •³ is not awarded. However, •⁴ is still available.

Commonly Observed Responses:

Candidate A - using addition formula $\int_0^{\frac{\pi}{9}} \left(\cos 3x \cos \frac{\pi}{6} + \sin 3x \sin \frac{\pi}{6} \right) dx$ $= \frac{1}{3} \sin 3x \times \frac{\sqrt{3}}{2} \dots \quad \bullet^1 \checkmark$ $\dots - \frac{1}{3} \cos 3x \times \frac{1}{2} \quad \bullet^2 \checkmark$	Candidate B - integrated over two lines $\int_0^{\frac{\pi}{9}} \left(\cos \left(3x - \frac{\pi}{6} \right) \right) dx$ $= \sin \left(3x - \frac{\pi}{6} \right) \quad \bullet^1 \checkmark$ $= \frac{1}{3} \sin \left(3x - \frac{\pi}{6} \right) \quad \bullet^2 \times$
Candidate C - integrated in part $3 \sin \left(3x - \frac{\pi}{6} \right) \quad \bullet^1 \checkmark \quad \bullet^2 \times$ $3 \sin \left(3 \times \frac{\pi}{9} - \frac{\pi}{6} \right) - 3 \sin \left(0 - \frac{\pi}{6} \right) \quad \bullet^3 \boxed{\checkmark 1}$ $3 \quad \bullet^4 \boxed{\checkmark 1}$	Candidate D - integrated in part $-\frac{1}{3} \sin \left(3x - \frac{\pi}{6} \right) \quad \bullet^1 \times \quad \bullet^2 \checkmark$ $-\frac{1}{3} \sin \left(3 \times \frac{\pi}{9} - \frac{\pi}{6} \right) + \frac{1}{3} \sin \left(0 - \frac{\pi}{6} \right) \quad \bullet^3 \boxed{\checkmark 1}$ $-\frac{1}{3} \quad \bullet^4 \boxed{\checkmark 1}$